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14. ABSTRACT   An algorithm for constructing the optimal local preconditioning matrix for 2-D hyperbolic systems was developed, applied to the equations of magnetogydrodynamics (MHD), and numerically tested. In addition, local preconditioners for the 1-D Navier- Stokes (N-S) equations were reviewed and the optimal N-S preconditioner was derived. (Local preconditioning reduces the local stiffness of equation systems caused by the range of time-scales of the physical processes described.) Numerical tests of the MHD preconditioner for MHD channel flow confirmed the convergence-acceleration effect and also the additional benefit of preserving solution accuracy for low-speed flow. For low-speed flow a simplified approximate preconditioner was formulated and tested. The optimal N-S preconditioner, as expected, renders the preconditioned equations unstable for certain unlikely combinations of low Mach and Reynolds numbers.

15. SUBJECT TERMS   Magnetohydrodynamics, Numerical Methods for hyperbolic equations, local preconditioning, multigrid methods, convergence acceleration, Navier-Stokes equations

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FINAL REPORT

Local preconditioning of the equations of magnetohydrodynamics and its numerical applications

AFOSR Grant Nr. F49620-00-1-0158

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Foreword

The funding period for the project "Local preconditioning of the equations of mag-netohydrodynamics and its numerical applications," AFOSR Grant Nr. F49620-00-1-0158, ended on October 31, 2002. A no-cost extension of 6 months was requested and granted, making the project closing date April 30, 2003. This is the Final Report for the entire project period, 2/1/00–4/30/03.

1   Stated objectives

The chief objective of the project was to develop a family of local preconditioning matrices for the three-dimensional equations of ideal magnetohydrodynamics (MHD) that reduce the stiffness of this system, caused by the spread among the MHD wave-speeds, down to its theoretical limit (yet unknown).

A second objective was to develop explicit preconditioned marching schemes for the MHD equations, geared toward convergence acceleration, and apply these to MHD-problems of interest to the Air Force. Such schemes may be used to march directly to steady MHD solutions, or as the inner iteration of a time-accurate implicit scheme when solving stiff unsteady MHD problems. Their explicit nature makes these schemes pre-eminently suited for scalable parallel computing. An additional benefit is accuracy preservation in the incompressible limit, important for magnetospheric and ionospheric applications.

A third objective was to modify preconditioning matrices for ideal MHD in such a way that they become effective for non-ideal MHD as well, i. e., including effects of resistivity, viscosity and heat conduction.

The development and application of local MHD preconditioning, never achieved

before, is expected to impact the entire MHD community, bringing efficient, scalable MHD calculations within everyone's reach.

2   Methodology used

Three different lines of research were persued, each with its own approach.

1. Rather than developing preconditioning for the special case of MHD equations, we developed a general construction method for optimal preconditioning matrices for arbitrary two-dimensional hyperbolic systems of conservation laws. This was a major achievement, with benefits extending to all disciplines in which wave phenomena are modelled.

   We then followed up with applying this technique to the MHD equations, constructing the unique optimal preconditioning matrix for these equations, a special form valid when the magnetic field is (close to) aligned with the flow, and an approximate form valid in the low-Mach number (almost incompressible) regime.

   The effect of these preconditioners on wave propagation (i.e., the clustering of wave speeds) was verified by an eigenvalue analysis of the preconditioned equations. Subsequently, the preconditioning matrix was incorporated in an explicit (multi-stage) second-order Godunov-type discretization of the MHD equations, and tested in two examples of MHD channel flow, with satisfactory results.

   On the negative side, we were not able, within the grant period, to come up with an algorithm for obtaining the optimal *three-dimensional* preconditioner, whether for a general hyperbolic system or just for the MHD equations. There appears to be a fundamental linear-algebra problem making the derivation impossible; all we can currently recommend is to try to extend the 2-D preconditioner to three dimensions using the specific physical content of each separate hyperbolic system.

2. A second line of research we followed was exploring the property of optimal local preconditioning to decompose a hyperbolic system into its time-asymptotically elliptic and hyperbolic parts, for the sake of convergence acceleration. To avoid delay this was done on the basis of the 2-D Euler equations, for which the optimal preconditioner was already known. We investigated multigrid cycles incorporating *streamwise semicoarsening* for the hyperbolic residual components, standard 2-D coarsening for the elliptical component, applied to a second-order Godunov-type discretization of the equations. As expected, we were able to get

convergence to steady subsonic flow in $O(N)$ operations. This technique is directly applicable to preconditioned MHD schemes and will greatly contribute to the efficient computation of steady plasma flows or plasma flows for which the use of an implicit time-marching technique is preferred (i. e., when the important time scales are considerable greater than those of faster but less important processes).

3. A third line of research was to search for an optimal preconditioner in case the system of equations includes dissipative terms. To avoid delay, we based this search on the 1-D Navier-Stokes equations. The successful search method was a combination of computational parameter studies and analytical interpretation of the numerical data. The resulting preconditioner does not immediately transfer to non-ideal MHD, nor to the multidimensional Navier-Stokes equations; more research is needed.

3   Accomplishments

1. A general method of constructing the optimal local preconditioning matrixfor 2-D hyperbolic systems was developed.

2. The optimal local preconditioner for the 2-D MHD equations was obtained.

3. The MHD preconditioner was simplified for use in two special cases:

   (a) magnetic field approximately aligned with the flow;

   (b) Mach number approaching zero (almost incompressible MHD).

4. For implementation in a Godunov-type finite-volume scheme, a construction algorithm for the artifical-viscosity matrix needed in such a scheme was developed.

5. Artificial viscosity matrices for the two simplified MHD cases were constructed and used in two computations of steady MHD channel flow. In both calculations the magnetic field was approximately aligned with the flow. In the first case the flow speed was chosen between the slow and fast magneto-acoustic wave speeds, in the second case a very low Mach number was chosen. Convergence to a steady flow was accelerated by factors 4 and 23, respectively. In the incompressible case the preconditioned solution was accurate, whereas the unpreconditioned solution was useless, as expected.

6. The property of optimal local preconditioning to decompose a hyperbolic system into its time-asymptotically elliptic and hyperbolic parts, was explored with regard to convergence acceleration. Multigrid cycles incorporating streamwise semicoarsening for the hyperbolic residual components, standard 2-D coarsening for the elliptical component, were applied to a second-order Godunov-type discretization of the 2-D Euler equations. In calculating steady subsonic flow over a bump, convergence to the steady solution was obtained in in $O(N)$ operations. Abandoning the streamwise semicoarsening increased the computational effort to $O(N^{\frac{3}{2}})$.

7. A perfect preconditioning for 1-D Navier-Stokes equations was derived by allowing all elements of the Euler (= high-Re) preconditioner to be modified as a function of Re and M as the Reynolds number drops below 1. Three distinct asymptotic regions were found requiring distinct preconditioning matrices. One region, defined by $M/Re \ll 1$, is problematic in that the optimal preconditioner destabilizes the equations by creating a growing mode. There is no way to avoid the growing mode without negating the preconditioning effect. In numerical calculations where this region happens to be visited, an implicit marching scheme or a multigrid strategy with sufficient damping is recommended.

8. All published local Navier-Stokes preconditioners were compared on the basis of a uniform set of state variables. There appear to be only three essentially different matrices (those due to Chorin, Turkel and Van Leer-Lee-Roe); all others reduce to these with minor modifications that do not affect performance.

4 Personnel

The following people took part in the research:

1. Bram van Leer, Professor, Principal Investigator;

2. Philip Roe, Professor, Co-Investigator;

3. Konstantin Kabin, postdoctoral fellow, May - August 2000;

4. Hiroaki Nishikawa, postdoctoral fellow, May 2001 - January 2003 (partial AF support in May - August 2001, non-AF support in January 2003);

5. Yoshihara Suzuki, graduate student, September 2002 - April 2003;

6. Christopher Depcik, graduate student, September 2001 - June 2003 (non-AF support).

5 Publications

During and slightly beyond the grant period the following conference papers were produced and presented:

1. H. Nishikawa and B. van Leer, "Optimal Multigrid Convergence by Hyperbolic/Elliptic Splitting," presented at 32nd AIAA Fluid Dynamics Conference, St. Louis, MO, 24-27 June 2002, AIAA Paper 2002-2951. *Prepared with 55% support from current grant.*

2. P. Roe and H. Nishikawa, " Toward a general theory of preconditioning," 32nd AIAA Fluid Dynamics Conference, St. Louis, MO, 24-27 June 2002, AIAA Paper 2002-2956.

3. H. Nishikawa, P. Roe, Y. Suzuki and B. van Leer, "A theory of local preconditioning and its application to 2D MHD equations," 16th AIAA Computational Fluid Dynamics Conference, Orlando, FL, 23-26 June 2003, AIAA Paper 3704-2003.

4. C. Depcik and B. van Leer, "In search of an optimal local Navier-Stokes preconditioner," 16th AIAA Computational Fluid Dynamics Conference, Orlando, FL, 23-26 June 2003, AIAA Paper 3703-2003.

6 Interactions/Transitions

(a) Phil Roe has advised the AF Research Laboratory, WPAFB (Drs. Miguel Visbal, Datta Gaitonde, Donald Paul) on issues of computational MHD and on establishing an AFRL Collaborative Center for Computational Science with our CFD group.

(b) There were no transitions; the research findings are not mature enough to give out of hands.

New discoveries, inventions, patents

None.